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Laser-Cooling in Noisy Quadrature of Squeezed Vacuum for Cesium Fountain Clock

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Abstract **In this paper we report that the noisy quadrature, a undesirable feature of the squeezed light, could be effectively used in realizing higher cooling force in the Cs fountain clock. We know that the laser cooling is basically the result of the combination of undesirable phenomena like light shift, Doppler shift and polarization gradient. We establish in this paper that the noisy quadrature, a unwelcome feature of the squeezed light is useful in achieving larger cooling force. When the atoms interact with the squeezed vacuum, they decay into two quadratures which respectively have quantum noise below and above the normal vacuum fluctuations. The noisy quadrature corresponds to the faster decay rate. On subjecting the Cs atoms to the noisy quadrature of the squeezed vacuum, superimposed with the squeezed coherent light, the cooling force is increased. The use of the noisy quadrature of the squeezed vacuum for obtaining larger cooling force and the temperatures much below the Doppler limit, without resorting to large detuning of the driving field, is a novel idea discussed in this paper. We report that the S/N of the clock signal may also be improved with the squeezed light.**

Keywords - **Cs fountain atomic clock, squeezed states, laser cooling force**

1 INTRODUCTION

The laser cooling and trapping of neutral atoms has opened up the way to the next generation of atomic frequency standards [1]. Many laboratories have already developed Cs fountain clock using laser-cooled atoms. The Cs atoms can be cooled to a few µKelvin temperature ($v_{rms} \sim 1$cm/s) either in the magneto-optic trap (MOT) or optical molasses. In the atomic fountain frequency standard, the atoms follow a parabolic trajectory for about one second, passing twice through the microwave cavity similar to the Ramsey separated rf field configuration. The long duration between the two microwave interactions leads to a sub-Hertz linewidth, which is two orders of magnitude narrower than that in the Cs thermal beam frequency standards. The laser-cooled Cs atomic frequency standard has stability of the order of 10^{-16} which is several orders better than the thermal beam Cs frequency standards.

In the laser-cooling set-up, the polarization gradient [2] and dark state [3] techniques are used for cooling the multi-level atoms to the temperatures much lower than the Doppler limit. However, even for the two-level atoms, very low temperatures can also be reached by using the squeezed light [4]. It has been shown that the temperatures slightly below the Doppler limit can be obtained [5] for the two-level atoms in a near-resonant standing-wave field. We establish in this paper that by using the noisy quadrature of the squeezed vacuum, the temperature much lower than the Doppler limit may be possible for the Cs atoms embedded in a squeezed vacuum. We shall see that the use of the noisy quadrature, which is the undesirable feature of the squeezed light, results in higher cooling force. It should not be surprising as the laser cooling is after all the combination of the undesirable phenomena of light shift, Doppler shift and polarization gradient. How well these undesirable aspects have helped in cooling the atoms to very low temperatures, is a established experimental fact.We unexpectedly have one more undesirable feature, that is the noisy quadrature in which the atoms decay rapidly on interaction with the squeezed vacuum.

2. SQUEEZED LIGHT COOLING OF ATOMS

In this paper, we study the cooling force on the Cs atoms in a Cs fountain frequency standard under the combined effect of the squeezed vacuum state and the squeezed coherent state. The squeezed vacuum and squeezed-coherent light form the optical molasses in which slow moving Cs atoms are embedded. The squeezed vacuum, with proper phase, may be made to interact with those atoms having enhanced dipole decay rate in the noisy quadrature. We discuss the interaction of the atoms with (i) coherent os of the squeezed vacuum in combination with the squeezed coherent light of proper phase the atoms can be cooled much below the Doppler limit even for a resonant driving field. It happens when the phase of squeezed vacuum relative to the laser field is 0 or π and the phase of squeezed coherent light is different from 0 or π..

We consider the Cs atomic fountain in which atoms are cooled in the presence of cw source of the squeezed light generated using an optical parametric oscillator (OPO). An external-cavity diode laser locked to Cs transition frequency at 852 nm is used as a primary source of light. The laser output at the wavelength 852 nm is sent to a frequency-doubling cavity for generating a blue pumping beam at 426 nm for the OPO. The doubling cavity contains a Potassium Niobate ($KNbO_3$) crystal as a nonlinear element and it is frequency locked to the incident laser. The output of the frequency doubler is used as a pump for the parametric down conversion in an OPO. The OPO cavity is a four-mirror folded cavity with two curved and two plain mirrors. The non-linear medium at the waist between curved mirrors is an a-cut $KNbO_3$ crystal. The spontaneous parametric fluorescence, produced by the degenerate down conversion into a sub-harmonic mode of OPO, leads to the squeezed vacuum. The reduction of quantum noise by 6 dB below the vacuum-state, for photo-current fluctuations in a balanced homodyne detector, has been demonstrated [6]. In a degenerate OPO, the pump field at 2ω is split by a non-linear crystal into two photons, each of frequency ω. The average values of photon annihilation and creation operators and their bilinear combinations in the squeezed vacuum, are $\langle a_k \rangle = \langle a_k^+ \rangle = 0$ [7] and $\langle a_k^+ a_k \rangle = N(\omega)$, $\langle a_k a_k \rangle = M(\omega) e^{i\phi}$ respectively. Here $N(\omega)$ is the photon number function and $M(\omega)$ is the two-photon correlation function. ϕ is the squeezed field phase. The photon number and the two-photon correlation functions are not independent of each other and satisfy the inequality $M^2(\omega) \le N(\omega)\,[N(\omega) + 1]$ in the quantum picture [8]. The factor (N+1) instead of N arises from the quantum nature of the field. For an ideal degenerate OPO [9], we have $N(\omega) = (\lambda^2-\mu^2)[1/(\omega^2+\mu^2) - 1/(\omega^2-\lambda^2)]/4$, $M(\omega) = (\lambda^2-\mu^2)[1/(\omega^2+\mu^2) + 1/(\omega^2-\lambda^2)]/4$, $M(\omega)^2 = N(\omega)\,[N(\omega) + 1]$, where $\lambda = \kappa/2 + \varepsilon$, $\mu = \kappa/2 - \varepsilon$. Here κ is the cavity decay constant and ε is the amplification factor for the OPO. The N and M functions may be selected by the parameters of the OPO. Such an OPO gives an ideal squeezed vacuum with $N = \sinh^2 r$, $|M| = \cosh r \sinh r$, here r is the squeezing factor. The signal and idler outputs of the OPO can be made to combine at a 50/50 beam-splitter to produce a squeezed-vacuum state and a squeezed-coherent state [10].

In the conventional laser-cooled Cs Fountain Frequency Standard, the Cs atoms are trapped by an optical molasses formed by three mutually perpendicular standing-wave laser fields. The cooling force due to the optical molasses has been calculated by several groups and is well-documented [11]. The force on slowly moving atoms with density matrix ρ, in a standing-wave squeezed-coherent field [12, 13] is given by

$$F = i\hbar q_r (\Omega^* \langle\rho_{12}\rangle - \Omega\langle\rho_{21}\rangle) =$$

$$\hbar q_r \Omega \langle\sigma_Y\rangle_{sc}, \qquad (1)$$

where $q_r = -k \tan(\mathbf{k}\cdot\mathbf{x})$, $k = 2\pi/\lambda$, $\lambda = 852$ nm, and Ω is the Rabi frequency of the laser field. $\langle\sigma_Y\rangle_{sc}$, the y-component of the atomic Block vector, in the presence of squeezed-coherent light, is given by

$$\langle\sigma_Y\rangle_{sc} = \Omega(\delta + \gamma|M|\sin\phi) / 2D_{sc}, \qquad (2)$$

where δ is the laser frequency detuning from the resonance, γ is the normal atomic decay rate, ϕ is the relative phase of the driving laser field with respect to the squeezed-coherent light,

$$D_{sc} =$$

$$n\left(\tfrac{1}{4}\gamma^2 n^2 + \delta^2 - \gamma^2|M|^2\right) + \Omega^2\left(n/_2 + |M|\cos\phi\right), \qquad (3)$$

and $n = 1 + 2N$. The force given by (1) is calculated assuming $\mathbf{k.v} < \gamma$ and $\delta < \gamma$, where $\mathbf{v}$ is the velocity of the atom. As is seen from (1) and (2), the magnitude of cooling force basically depends on $(\delta + \gamma|M|\sin\phi)$, giving rise to cooling force even at zero detunings. In the cooling force (1) and (2), the term $\gamma|M|\sin\phi$ arises due to the interaction of atoms with the squeezed-coherent light. Thus we observe that the squeezed light enhances the cooling force. But even with this, atoms do not attain very low temperature near resonance.

3.COOLING FORCE ENHANCEMENT IN NOISY QUADRATURE

For a given value of the detuning, the cooling force may be increased by optimizing the factor $\gamma|M|\sin\phi$. Here M, the two-photon correlation factor, depends on the degree of squeezing. Besides, the decay rate γ is also an important parameter in deciding the cooling force. We shall see below that the increased decay rate results in a larger cooling force. It is well established that the squeezed states can manipulate the decay rate [9,10]. When the atoms are placed in the squeezed vacuum, they decay in two quadratures with different decay rates. In the squeezed (in-phase) quadrature, the atom decays at a rate slower while in the noisy (out-of-phase) quadrature it decays at a rate faster than in the normal vacuum respectively. Specifically, when the relative phase Φ between the coherent driving field and the squeezed vacuum is 0 or π, the atoms decay in out-of-phase or the noisy quadrature with the decay rate $\gamma_x = \gamma(N + \tfrac{1}{2} + M)$. In general, the squeezed states are used for reducing the quantum noise or the decay rate of the atom. However, in this paper, we demonstrate that even the noisy quadrature could be gainfully utilized in realizing the higher cooling force.

The atoms, embedded in the noisy quadrature of the squeezed vacuum having faster decay rate γ_x, are allowed to interact with the squeezed-coherent light of the phase ϕ other than 0 or π. We now calculate the cooling force on the atoms in the combined presence of squeezed vacuum and squeezed coherent states. Incorporating the faster decay rate of atoms in (1), the cooling force is

$$F_{sv} = q_r \Omega \langle \sigma_Y \rangle_{svsc}, \qquad (4)$$

here

$$\langle \sigma_Y \rangle_{svsc} =$$

$${}^1/_2 \beta \, [\Delta + 2|M| \, (n/2 + |M|) \sin\phi] / D, \qquad (5)$$

$$D =$$

$$n/_2 \, [\beta^2 + 2\Delta^2 + (n/2 + |M|)^2] + \beta^2 |M| \cos\phi. \qquad (6)$$

The normalized detuning Δ and Rabi frequency β are defined as $\Delta \equiv \delta/\gamma$, $\beta \equiv \Omega/\gamma$. It may be readily shown that with the modified y-component of the atomic Block vector, $\langle \sigma_Y \rangle_{svsc}$, the cooling force is increased depending on the phase ϕ and squeeze parameter r. For the different values of these parameters the cooling force is calculated.

The relevant parameter quantifying the squeezing is the degree of squeezing which is given by the factor $2(|M| - N)$. Georgiades, et. al [14] have developed a source of squeezed light exhibiting approximately 75% squeezing. This corresponds to $N = 9/16$, $|M| = 15/16$ and $r \approx 0.7$. For 75% squeezing we compare the cooling force F and F_{sv}. As shown in Fig. 1a and 1b, for small values of Rabi frequency β; F (dashed line) is greater than F_{sv} (shown by solid curve). For $\beta > 3$, this trend changes. There is a steep rise in F_{sv} compared to F with the increasing values of β.

This shows the utility of the noisy quadrature over the in-phase squeezed quadrature for the squeezed light cooling of the atoms. In Fig. 2a and 2b, we show that the relative phase ϕ and the degree of squeezing play significant role in lowering the temperature in the noisy quadrature. Figs. 2(a) and 2(b) are for the resonant and off- resonant cases respectively. These figures show that cooling force is sensitive to the small changes in relative phase between fields, the detuning, the magnitude of Rabi frequency and degree of squeezing. The atoms in presence of squeezed vacuum, when its phase Φ relative to the laser field is 0 or π, can attain minimum temperature much lower than that given by Doppler limit $k_B T_D = h\gamma/2$. Here k_B is the Boltzman constant.

Fig. 1(a) is a plot for the resonant case. We find that the squeezing can give rise to a cooling force even at zero detunings. In Fig 1(b), we plot the off-resonance case. This shows the similar behavior as in Fig. 1(a) except that the cooling force increases slightly. The use of the squeezed (in-phase) quadrature instead of noisy quadrature (out of phase) by selecting the phase $\Phi = \pi/2$ will make the cooling force (shown by dotted line in Fig. 1) much smaller than F (shown by dashed line).

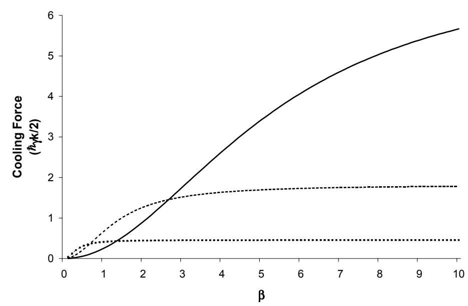

Fig. 1 (a) resonance case $\Delta = 0$ The dependence of spatially averaged force on the Rabi frequency β of the driving field in the presence (solid and dotted line) and absence (dashed line) of squeezed vacuum. The solid line is for the phase $\Phi = 0$ or π (out-of-the phase) and dotted line is for phase $\Phi = \pi/2$ (in-phase quadrature). Here degree of squeezing is chosen to be 75% and $\phi = 0.8\,\pi$.

4. SIGNAL TO NOISE RATIO (S/N)

The frequency stability of the laser cooled Cs fountain frequency standard is given by

$$\sigma(\tau) = \frac{\delta\upsilon (Tcycle/\tau)^{1/2}}{\pi\upsilon \, (S/N)} \qquad (7)$$

here υ is the microwave clock transition frequency (9.192 GHz), $\delta\upsilon$ is its linewidth, τ is the sampling time and S/N is the clock signal to noise ratio and Tcycle is the total loading and fountain cycle. With technical noise under control, the quantum noise may be suppressed using squeezed light. The S/N with squeezed light [6] may be expressed as $S/N_{SQ} = \eta\Lambda_{SQ}^2/[2\{1 + \xi \, S(\Omega_0, \varphi_-)\}]$. η is related to the detection efficiency of the vacuum fluctuations, Λ_{SQ} is the response of the atoms to the squeezed field. ξ is the efficiency of propagation of the squeezed field and $S(\Omega_0, \varphi_-)$ is the spectrum of the squeezing for the reduced (quadrature) fluctuations of the detection light. $S(\Omega_0, \varphi_-)$ tends to -1 as the degree of squeezing is increased and it reduces to zero for coherent light Thus with the squeezed light as the

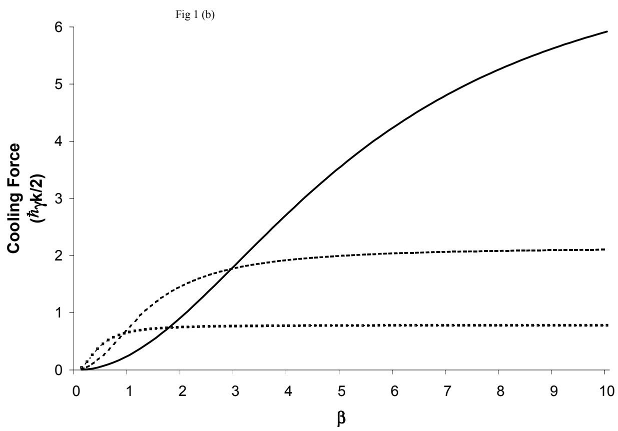

Fig.1(b) Off resonance case case $\Delta \neq 0$ The dependence of spetially averaged force on the Rabi frequency β of the driving field in the presence (solid and dotted line) and absence (dashed line) of squeezed vacuum.The solid line is for the phase $\Phi = 0$ or π (out-of-the phase) and dotted line is for phase $\Phi = \pi/2$ (in-phase quadrature). Here degree of squeezing is chosen to be 75% and $\phi = 0.8\pi$.

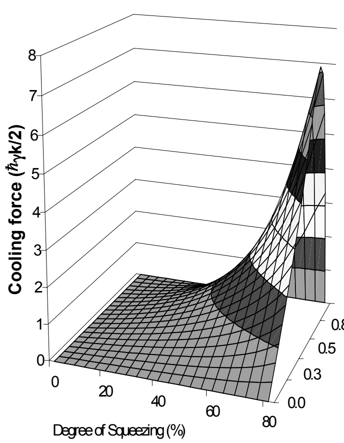

Fig.2(a) A three-dimensional plot of spatially averaged cooling force Fsv as a function of the degree of squeezing and relative phase ϕ for the resonant case $\Delta = 0$.

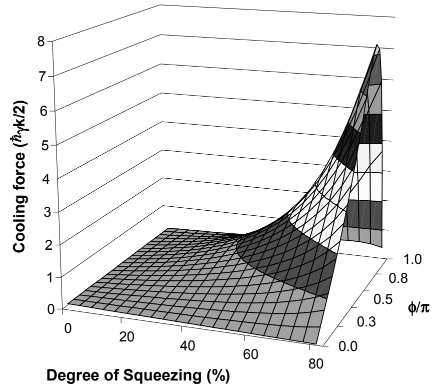

Fig2(b) A three-dimensional plot of spatially averaged cooling force F_{sv} as a unction of the degree of squeezing and relative phase ϕ for the off-resonant case $\Delta = 0.1$. Rabi frequency $\beta = 10$.

probe, the S/N of the clock signal may be improved and it depends on the degree of squeezing. It may lead to a better frequency stability of the Cesium Fountain Clock .

CONCLUSION

It has been shown that Cs atom can have super-natural and sub-natural linewidth [7-9], both of which can be used to lower the temperature of the trapped atoms. Interestingly, due to the enhanced spontaneous emission rate of the optical transition of Cs in the presence of the squeezed vacuum field of proper phase, along with the squeezed-coherent field, the cooling force is increased and very low temperature may be reached. With the certain values of the phase of the squeezed coherent field with respect to the laser field and for appropriate two-photon correlation M, atoms can be cooled even for the resonant laser light. We have earlier shown [15] that the frequency stability and the accuracy of Cs Beam Frequency Standards can be improved by using squeezed states for the selection of the atomic state as well as for detection of the clock transition. This is because, the narrowing of the lineshape of the fluorescence spectrum with the application of the squeezed states results in higher S/N. Similarly, the use of squeezed vacuum with laser light for state selection and detection in the Cs Fountain clock will result in better frequency stability. As the squeezed light may create the atomic correlation [16] the atomic spin projection noise can be controlled through the squeezed states.

The vacuum fluctuations restrict the minimum temperature, determined by the Doppler Limit $k_B T_D = h\gamma/2$. As shown in Fig. 2, even for the small degree of squeezing ($\geq 40\%$) and certain values of relative phase, atoms can be cooled much below the Doppler Limit. Further one can also calculate the quantum fluctuations experienced by the atom [5].

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